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2D Spiking Deconvolution Approach to Resolution Enhancement of Prestack Depth Migrated Seismic Images

A.K. Takahata* (University of Campinas), L.J. Gelius (University of Oslo), R. R. Lopes (University of Campinas), M. Tygel (University of Campinas) & I. Lecomte (NORSAR)

SUMMARY

Complex velocity models, limitations in acquisition geometry and frequency bandwidth, give rise to distortions in prestack depth migrated (PSDM) images. Such distortions can be modelled as the 2D convolution between the actual reflectivity and a resolution function. In the case of Born scattering, the resolution function is referred to as point spread function (PSF). The PSFs can be calculated with relatively low computational effort by ray tracing. In this work, we review the basic idea of the PSF and its relationship with seismic images generated by PSDM. With the help of the PSF concept, we propose the use of 2D spiking deconvolution with the aim of minimizing these image distortions. Finally, the potential and limitations of the proposed method are explored with applications on controlled synthetic data.

Introduction

Geologic complexities of the subsurface and limitations in acquisition geometry and frequency bandwidth, among other factors, lead to reduced resolution in prestack depth migrated (PSDM) images. Under the assumption of Born scattering, the resulting distortion may be modelled by the so-called *resolution functions* or *point spread functions* (PSFs) (Lecomte and Gelius, 1998; Gelius and Lecomte, 2000; Gelius et al., 2002). These PSFs can be calculated with relatively low computational effort by means of ray tracing and can be used to enhance the resolution of PSDM images through 2D deconvolution methods (Gelius et al., 2002; Sjoeberg et al., 2003). However, these works only consider a smaller subsection along the vertical direction of each PSF (pseudo 2D). Other approaches, such as, least-squares migration (Nemeth et al. 1999) and migration deconvolution (Hu et al., 2001), consider the whole modelling operator and its adjoint (migration operator), but require a larger computational effort or simplifying schemes, e.g., consideration of the main diagonal of the Hessian matrix only (see discussion in, e.g., Tang, 2009). In the present work, we propose an alternative approach based on 2D regularized filtering and spiking deconvolution. This contrasts with previous works, such as (Sjoeberg et al., 2003), in which an inversion approach was used. We present results based on controlled data, and discuss the potentials and limitations of the method.

The resolution function and point spread functions

Let $x(\mathbf{r})$ be a function of subsurface position $\mathbf{r}$ that refers to a geological model, say, the reflectivity, at point $\mathbf{r}$. Following (Gelius et al., 2002; Sjoeberg et al., 2003; Lecomte, 2008), we assume that the acquisition geometry, the background velocity model and the source signature are known. Given a model point $\mathbf{r}$, consider the local spatial Fourier transform of $x(\mathbf{r})$ at a small region around $\mathbf{r}$, denoted by $X(\mathbf{K})$. The Fourier vector $\mathbf{K}$ is the *scattering wavenumber vector* and it relates to the seismic survey geometry and the signal frequency spectrum and the velocity model, as follows

$$\mathbf{K} = \omega(-\mathbf{p}_s + \mathbf{p}_g)\,. \qquad (1)$$

Here, $\mathbf{p}_s$ and $\mathbf{p}_g$ are the slowness vectors, which are parallel to the incident and scattered rays at $\mathbf{r}$. A perfect estimation of $x(\mathbf{r})$ requires a 360$^{\mathrm{o}}$ coverage of $\mathbf{K}$ vectors (thus requiring both scattered and transmitted waves), with their magnitudes ranging from zero to infinity (thus requiring an infinite signal frequency band). In practice, this is not possible, since seismic surveys are limited to a finite number of sources and receivers, whose positions are generally constrained to an acquisition surface. The velocity model also plays a major role, as it limits the orientation, coverage and size of the slowness vectors. Finally, the frequency band is constrained by the seismic signature bandwidth. Such limitations can be modelled in the $\mathbf{K}$ domain by a function, $H(\mathbf{K})$, in which ray-tracing techniques can estimate the orientation and coverage, and the source signature (extracted from the data) can estimate the size of $\mathbf{K}$ (Gelius et al., 2002; Lecomte, 2008). The function H($\mathbf{K}$) is related to the estimate of the desired quantity, y($\mathbf{r}$), in our case the PSDM image, by

$$y(\mathbf{r}) = \int \mathrm{H}(\mathbf{K})X(\mathbf{K})\exp(j\mathbf{K}\cdot\mathbf{r})\mathrm{d}\mathbf{K} = \int x(\mathbf{r}')h(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathbf{r}'\,. \qquad (2)$$

The second integral expresses the product in the Fourier domain (first integral) as a convolution in the space domain. In this case, $h(\mathbf{r})$ is the *resolution function* being, in fact, the inverse Fourier transform of $H(\mathbf{K})$. In the Born scattering model, it is also called the *point spread function* (PSF) and it models how much the image of a model point has been distorted. Note that the convolution model presented in Eq. (2) is valid only if the PSF is space invariant. This requires, from Eq. (1), that all subsurface points have the same illumination, which is a valid approximation if we consider a small target area below a smooth overburden.

2D spiking deconvolution

In order to minimize the impacts of the limited seismic acquisition and the velocity model, we propose the use of a 2D spiking deconvolution strategy. This is analogous to the 1D case, in which a deconvolution filter is calculated in order to transform the actual source signature into a narrow wavelet (ideally a spike), thus enhancing the vertical resolution. In the 2D case, a wide PSF, $h(m,n)$, represented by $(2M+1)\times(2N+1)$ pixels (Fig. 1a), is filtered with a $(2P+1)\times(2Q+1)$ 2D filter, $w(p,q)$, (Fig. 1b), in order to produce an output, $c(m,n)$, which is a narrower PSF (Fig. 1c). In other words, we want that

$$c(m,n)=\sum_{p=-P}^{P}\sum_{q=-Q}^{Q}h(m-p,n-q)w(p,q)\approx\delta(m,n)\,, \tag{3}$$

where $\delta(m,n)$ represents an ideal spike, which has unit value at the origin and zero at the remaining points (Fig. 1d). Actually, the equality is not attainable in practice, since it would require a deconvolution filter of infinite size, as in the 1D case. Note that a general image resulting from the blurring described by Eq. (2) can be seen as a linear superposition of several PSFs, each one of them acting around a point of the original image, weighted by its amplitude. Thus, the 2D spiking deconvolution filter will work by narrowing each of these shifted and scaled PSFs, thereby enhancing the overall resolution of the image.

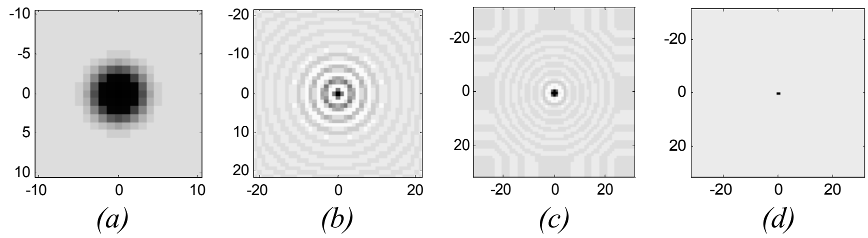


Figure 1 *(a) PSF. (b) 2D Deconvolution filter. (c) Deconvolved PSF. (d) Ideal spike.*

As seen below, our proposed algorithm can be conveniently described by the use of the so-called lexicographic ordering as in Sjoeberg et al. (2003) to represent 2D images as 1D vectors. Figs. 2a and 2b show an example for a P=1 and Q=1 image, $h(p,q)$, and its vector representation, respectively. First, notice that in Eq. (3), $h(m-p,n-q)$ describes a flipped and shifted version of $h(p,q)$ (Fig. 2c for m=n=0 and Fig. 2d for m=n=1). In Fig. 2d, the white spaces represent null elements. If we denote $\mathbf{h}^{m,n}$ as the lexicographical representation of $h(m-p,n-q)$, as in Fig 2e, and $\mathbf{w}$, as the corresponding vector for $w(p,q)$, we may rewrite Eq. (3) as $c(m,n)=(\mathbf{h}^{m,n})^T\mathbf{w}$. Furthermore, if we denote $\mathbf{c}$ as the lexicographical representation of $c(m,n)$, then we have $\mathbf{c}=\mathbf{Hw}$, where each row of $\mathbf{H}$ corresponds to $(\mathbf{h}^{m,n})^T$. An example is shown in Fig. 2f for M=1 and N=1. Note that Fig. 2e corresponds to the seventh row of Fig. 2f. If we represent the lexicographic representation of $\delta(m,n)$ as $\boldsymbol{\delta}$ and the regularization (prewhitening) parameter as λ, then, the optimal least squares filter is given by

$$\mathbf{w}=(\mathbf{H}^T\mathbf{H}+\lambda\mathbf{I})^{-1}\mathbf{H}^T\boldsymbol{\delta}\,, \tag{4}$$

analogously to the 1D case (for further discussion on the 1D case see, e,g., Treitel and Lines (1982)).

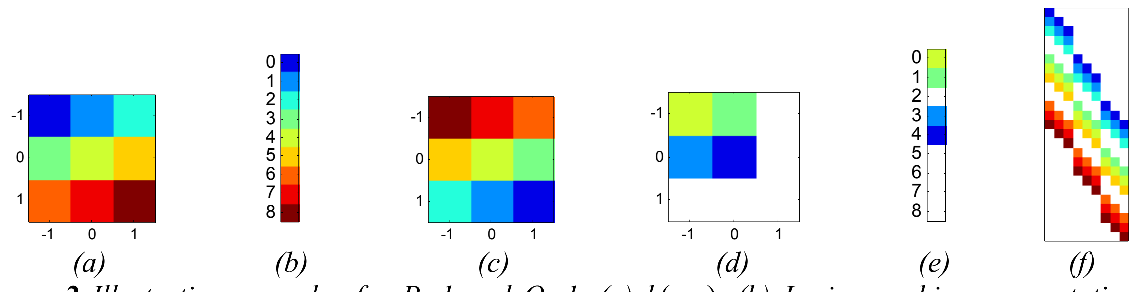
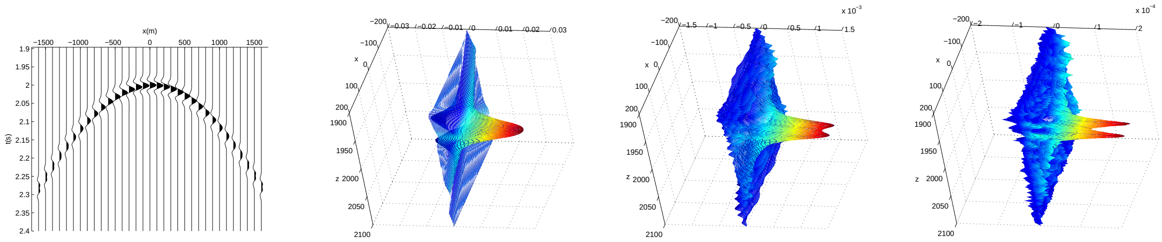


Figure 2 *Illustrative examples for P=1 and Q=1. (a) $h(p,q)$. (b) Lexicographic representation of $h(p,q)$. (c) $h(-p,-q)$. (d) $h(-1-p,-1-q)$. (e) $\mathbf{h}^{-1,-1}$. (f) Matrix $\mathbf{H}$ for M=1 and N=1.*

Controlled data results

As an example, consider two nearby scatterers in a homogeneous model with velocity C=2000m/s. They are located at $(-20, 2000)^T$ and $(30, 2000)^T$, with the coordinate system given by $(x, z)^T$, so that they are diffraction limited. A common shot gather is shown in Fig. 3a, with source at the origin and receivers ranging from $(-1600, 0)^T$ to $(1600, 0)^T$ with a spacing of 10m. The source gather was migrated with diffraction stacking (Fig. 3b). As expected, migration cannot resolve the two scatterers.

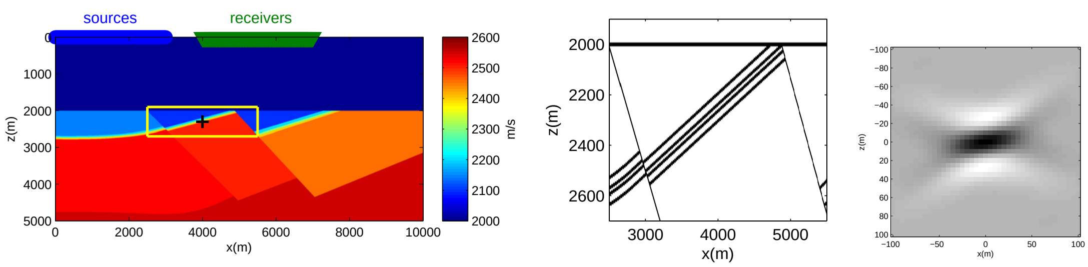
However, if the PSDM image is subjected to the 2D deconvolution filter proposed in this paper, the two scatterers can be resolved due to increased spatial resolution. This is shown in Fig. 3d. The 1D (pseudo 2D) deconvolution was also tested by considering just the central vertical part of the PSF, as in Gelius et al. (2002), Fig. 3c. In this case, the lateral resolution is slightly increased, but the 2D deconvolution provides a better separation between the scatterers, as expected.



(a) *(b)* *(c)* *(d)*

Figure 3 *(a) Common shot gather. (b) Migration. (c) 1D deconvolution. (d) 2D deconvolution.*

Next, we consider a fault model with several thinner layers as shown in Fig. 4a. A common-offset section (offset 4000m) was computed using 2D ray tracing. The diffraction-stack migration for the small target selected in Fig. 4a is shown in Fig. 5a. From the four inclined reflectors displayed in Fig. 4b, only three of them are now visible. Next, the 2D deconvolution method was applied to the migrated image, as shown in Fig. 5b, using the PSF (Fig. 4c) calculated at the point indicated by the black cross in Fig. 4a. Note that all four reflectors were now resolved. However, it is also observed that the original migration artefacts in Fig. 5a have been somewhat amplified. Also, some ringing is associated with the reflectors. This can be partly explained by the fact that, by an analogy to the 1D case, the 2D spiking deconvolution filter calculated in Eq. (4) is, indeed, the least-squares approximation to the inverse filter of the PSF. This implies that the amplitude spectrum of the spiking deconvolution filter has high values associated to the positions in the **K** domain whose amplitudes are low in the PSF, as Figs. 6a and 6b show. Actually, Eq. (1) indicates that the PSF has vanishing small amplitudes associated with **K** values that represent frequencies outside the band of the source signature or directions with no illumination, i.e., components that do not actually exist in the seismic image. Thus, the enhancement of these components results in the artefacts (Fig. 5b). In order to eliminate them, we use a 2D filter calculated from a tapered **K** domain mask (Fig. 6c), so that only non-zero amplitudes of the components of the PSF in the **K** domain are allowed to pass. Fig. 5c shows the result, where the artefacts have been removed while keeping the resolution enhancement.

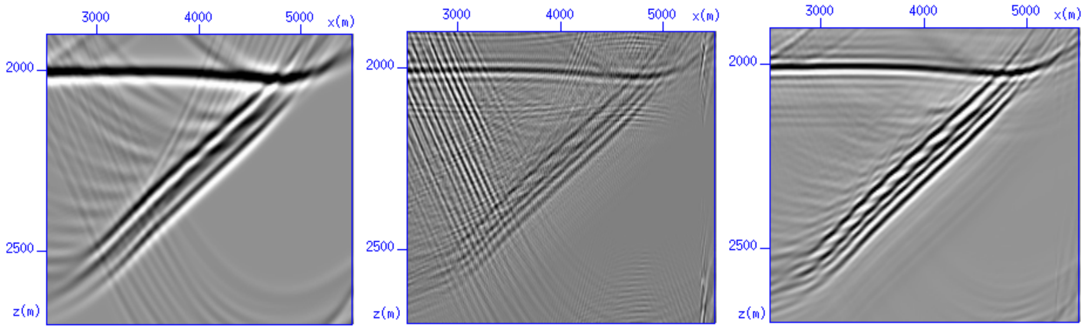


(a) *(b)* *(c)*

Figure 4 *(a) Fault model and target. (b) Reflectivity section. (c) PSF at the black cross.*

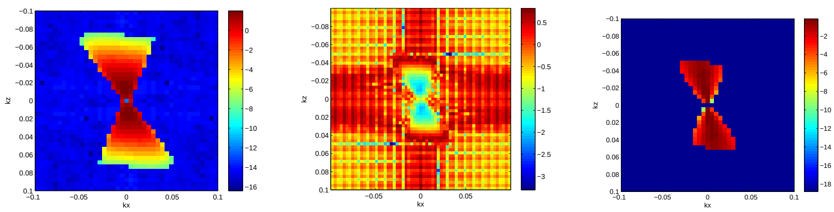
Conclusions

This work reviews the concept of resolution functions and PSFs and describes their role in seismic imaging through the introduction of a convolutional blurring model. Next, the use of 2D regularized spiking deconvolution was proposed in order to deblur a PSDM image. This concept was tested by applying the method to migrated synthetic data. In case of a homogeneous medium it was demonstrated that the use of 2D deconvolution improves the lateral resolution of migrated images. The results obtained in case of a layered fault model showed that the method is also able to enhance the resolution of reflectors. However, already existing migration artefacts can possibly be further enhanced by the deconvolution if that is not done with care. A refinement was proposed with the implementation of a 2D filter based on a tapered 2D wavenumber mask. Further improvements could include the use of a locally determined *reflector spread function* (Kirchhoff type) instead of the Born or point spread type of resolution function (Gelius et al., 2002).



(a) *(b)* *(c)*

Figure 5 *(a) Migrated image. (c) 2D deconvolution. (c) 2D deconvolution + Filtering.*



(a) *(b)* *(c)*

Figure 6 *Amplitude* **K** *spectra: (a) PSF. (b) 2D deconvolution filter. (c) Tapered wavenumber mask.*

Acknowledgements

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